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### Non-cooperative Games

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**NON-COOPERATIVE GAMES**

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# NON-COOPERATIVE GAMES

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## ABSTRACT

Non-cooperative games are mathematical models of interactive strategic decision situations. In contrast to cooperative models, they build on the assumption that all possibilities for commitment and contract have been incorporated in the rules of the game. This contribution describes the main models (games in normal form, and games in extensive form), as well as the main concepts that have been proposed to solve these games. Solution concepts predict the outcomes that might arise when the game is played by “rational” individuals, or after learning processes have converged. Most of these solution concepts are variations of the equilibrium concept that was proposed by John Nash in the 1950s, a Nash equilibrium being a combination of strategies such that no player can improve his payoff by deviating unilaterally. The paper also discusses the justifications of these concepts and concludes with remarks about the applicability of game theory in contexts where players are less than fully rational.

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## NON-COOPERATIVE GAMES

Games are mathematical models of interactive strategic decision situations: there are various actors (players) involved that jointly determine the outcome and each tries to obtain that outcome that is most favorable to him. A game is said to be non-cooperative if there are no possibilities for commitment (unilateral or multilateral) outside the rules of the game. In contrast, in cooperative games, players can form coalitions with the possibilities for doing so not being explicitly modeled within the rules of the game. Nash (1953) argued that the two approaches are complementary and he proposed building non-cooperative models of cooperative games, an idea that is referred to as the Nash program. See the entry on COOPERATIVE GAMES for more details. Below non-cooperative game models and solution concepts are described and some applications are indicated. See Aumann (1987) for a more extended overview.

### THE PROBLEM

Imagine an interactive decision situation in which various individuals (players) are involved, whose decisions affect each other. Depending on the decisions that are taken, a certain outcome will result; players have preferences over these outcomes and typically there is a conflict of interest as different players prefer different outcomes. Each player now faces the question what decision is best for him. As the decisions are interdependent, the answer to the question will depend not only on the player's own preferences, but also on the decisions that the other players take. The player, hence, has to make a prediction about what the other players will do. For an outsider there is the related question of predicting the overall outcome of the game. Based on the assumption of rational behavior of all players involved, game theory offers a set of tools and concepts, that provide answers to these questions.

The above mentioned problem arises in a variety of circumstances, ranging from parlor games (chess, poker, bridge) to various economic, political, military or biological situations. Game theory offers a collection of formal models and solution concepts to analyze these situations. While the history of the field can be traced back to the analysis of the two-person card game "le Her" by James Waldegrave in the early 18th century, with important contributions by the French scientists Cournot and Borel in the 19th and 20th century (Baumol and Goldfeld (1968), Weintraub (1992)), John von Neumann is generally regarded as the founder of the field. In Von Neumann (1928), he showed that the above mentioned problem allows a solution in the case of two players with strictly opposed interests (zero-sum games). The book "Theory of Games and Economic Behavior" that von Neumann wrote together with the economist Oskar Morgenstern (Von Neumann and Morgenstern (1944)) convincingly demonstrated that the tools initially developed for parlor games could be successfully applied to a variety of social conflict situations.

## NON-COOPERATIVE GAME MODELS

The simplest type of model is one in which there is no dynamics: each player has to make only one decision and decisions are effectively made simultaneously as no player has knowledge about the decision that has been made by another player. As an example, one might think of a sealed bid procurement auction: without knowing the bids of the competitors, each player submits his bid in a sealed envelope; the player making the lowest bid is awarded the contract and in turn receives a payment (say) equal to his bid. Such a game is called a game in normal form, or also, a game in strategic form.

The formal mathematical definition of an  $n$ -player *normal form game* is as follows. Let  $S_i$  be the (finite) set of possible decisions (also called actions or strategies) that player  $i$  might take. Furthermore, let  $S = \prod_{i=1}^n S_i$  be the set of strategy profiles, hence,  $s \in S$  specifies a strategy for each and every player in the game. Assume that each player's preferences over the set of outcomes of the game can be described by a (Von Neumann Morgenstern) utility function, hence, each player wants to maximize his utility and he only cares about expected utility. (Von Neumann and Morgenstern (1953) give conditions under which such a utility function can be found; some theory has been developed also without this assumption, but it is convenient to maintain it here). Each strategy profile  $s \in S$  produces a certain outcome and write  $u_i(s)$  for the utility of player  $i$  associated with this outcome. The normal form game is then completely specified by the strategy sets  $S_i$  and by the utility functions  $u_i$  with  $u_i : S \rightarrow \mathbb{R}$ , hence  $G = \langle S_1, \dots, S_n, u_1, \dots, u_n \rangle$ .

Games in which players move more than once, and/or in which moves of different players are sequential can be represented by means of a tree. Formally, such a game is said to be in *extensive form*. The nodes of the tree correspond with decision points of players, with the arcs at a node representing the decisions that are possible at that node. Each endpoint of the tree represents an outcome of the game and again it is associated with a utility for each of the players. A special type of extensive form game is one in which there is perfect information: when a player has to move, he has full information about where he is in the game and what decisions have been taken before. Chess can be viewed as an extensive form game with perfect information. So called information sets are used to represent limited information in a tree: when a player cannot distinguish two nodes in the tree, these are in the same information set. When a player has to move, he can condition his action only on the information set that is revealed to him, not on the actual node. The theory has been mainly developed for games with *perfect recall*, that is, each player is assumed to fully remember his previous actions and information. Kuhn (1953) provides a formal development. For some recent contributions dealing with imperfect recall, see Rubinstein (1998).

Von Neumann (1928) introduced the fundamental concept of *strategy* by means of which

an extensive form game can be reduced to one in normal form. Formally, a strategy for a player is a full plan of action that specifies for each information set of this player the action that this player intends to take if and when that information set is reached. Clearly, once a strategy has been determined for each player, the outcome is determined as well. Von Neumann argued that, for rational players, there is no loss of generality in forcing them to think through the game in advance, hence, to force them to choose a strategy. Consequently, the extensive form can be reduced to the simpler normal form.

The above description abstracted from randomization. A player, however, might randomize before the start of the game which (pure) strategy he might use during the game. Alternatively, he might randomize to determine which action he will choose once a certain information set is reached. The concept of *mixed strategy* is used to refer to the former case, the concept of *behavior strategy* refers to the latter case of local randomization. Kuhn (1953) showed that both concepts are equivalent for games with perfect recall, that is, whatever a player can do with a strategy of one type he can also do with a strategy of the other, and vice versa. Note that since players are assumed to have Von Neumann Morgenstern utilities, replacing "payoff" (utility) by "expected payoff" creates no special difficulties.

The traditional theory has been developed for games of complete information, that is, each player is assumed to know the utility functions of all the players in the game. Frequently, a player will not have all information about his competitors. For example, in the procurement context referred to above, a player may not know how many orders a competitor has in stock, hence, he may not know how much value an opponent assigns to winning the present contract. Harsanyi (1968) showed how to incorporate incomplete information into the model. To represent asymmetric information, Harsanyi introduces an artificial chance move at the beginning of the game that determines which piece of private information each player will have. This piece of private information is also called the player's type. It is assumed that the set of possible types and the probability distribution over this set is common knowledge, however, which type realization results for player  $i$  is typically only known to player  $i$  himself. A *normal form game with incomplete information*, hence, does not only specify the players and their utility functions, but also their types and the joint probability distribution on players' types. Formally such a game is given by a tuple  $G = \langle S_1, \dots, S_n, T_1, \dots, T_n, u_1, \dots, u_n, p \rangle$  where  $S_i$  is player  $i$ 's strategy set,  $T_i$  is the set of possible types of player  $i$ ,  $u_i: S \times T \rightarrow \mathbb{R}$  is player  $i$ 's payoff function (where  $T = \prod_{i=1}^n T_i$ ) and  $p$  is the probability distribution on  $T$ . The play of the game proceeds as follows: A type profile  $t \in T$  is determined according to  $p$  and player  $i$  is informed about his type  $t_i$ . Based on this information, player  $i$  updates his information about the types of the others, computing the posterior probability  $p_i(t/t_i)$  and he chooses an action to maximize his associated expected payoff. Harsanyi's modeling technique has shown to be very powerful. See the entries on AUCTIONS and ECONOMICS OF INFORMATION for examples.

## SOLUTION CONCEPTS

### 1. Individual Rationality

Concepts of "individual rationality" from decision theory can also be used in a game context. If one strategy  $s_i$  of player  $i$  is (strictly) *dominated* by another strategy  $s_i'$  of this player (i.e. it yields a lower payoff no matter what strategy profile  $s_{-i}$  the other players use, formally  $u_i(s_i, s_{-i}) <$

$u_i(s_i', s_{-i})$  for all  $s_{-i}$ ) then a rational player  $i$  will not use  $s_i$  and it should be possible to eliminate  $s_i$  without changing the solution. Furthermore, dominated strategies can be iteratively eliminated. Alternatively, a "Bayesian" player will represent his uncertainty about his opponent's strategies by a probability distribution  $\sigma_{-i}$  on  $S_{-i} (= \prod_{j \neq i} S_j)$  and he will thus only choose strategies that are *best responses* against some such correlated strategy  $\sigma_{-i}$ . By using duality theory from linear programming it can be shown that  $s_i$  is a best response against some correlated strategy  $\sigma_{-i}$  of the opponents if and only if  $s_i$  is not strictly dominated, hence, the two concepts are equivalent. In most of the theory it has been assumed that the fact that players decide independently implies that a player will represent his uncertainty by a profile of mixed strategies of the opponents, that is, the components of  $\sigma_{-i}$  are independent. The set of strategy profiles that remain after all strategies of all players that are not best responses against independent mixtures of opponents have been iteratively eliminated is called the set of *rationalizable profiles* (Bernheim (1984), Pearce (1984)). The set of rationalizable strategies may be smaller than the set of iteratively undominated strategies.

In his theory of the 2-person zero-sum game, Von Neumann also used an individualistic rationality concept. If player  $i$  uses mixed strategy  $\sigma_i$  then the worst that can happen to him is to receive the expected payoff  $\min_{\sigma_{-i}} u_i(\sigma_i, \sigma_{-i})$ , hence, player  $i$  can guarantee himself the payoff  $\max_{\sigma_i} \min_{\sigma_{-i}} u_i(\sigma_i, \sigma_{-i})$  by choosing his mixed strategy appropriately. Note that a mixed strategy may "guarantee" a higher expected payoff than a pure strategy. For example, in "matching pennies" (two players simultaneously show the side of their penny with player 1 (resp. player 2) receiving both pennies if the sides are the same (resp. different) it is obviously better to keep one's side hidden, hence, to use a mixed strategy. Von Neumann showed that, for finite 2-person zero-sum games, there exists a numbers  $v_1$  and  $v_2$  with  $v_1 + v_2 = 0$  such that each player  $i$  can guarantee himself  $v_i$ . We say that  $v_i$  is the *security level* of player  $i$ : against a "rational" opponent, player  $i$  cannot expect to get more than this level. Strategies that guarantee the security level are called *optimal strategies* and  $v_1$  is also termed the *value of the game*. Von Neumann's theorem is also called the *minmax theorem*, it thus shows that there is indeed an optimal way to play a finite 2-person zero-sum game.

### 2. Equilibrium

John Nash (1950, 1951) was the first to break away from the individualistic rationality concepts. Rather than focusing on the direct question "what should a player do?" (or "what should a game theorist advise a player to do?"), he approached the problem indirectly and asked "if the game has a solution, what properties would it have?" In fact, a suggestion to proceed in this way was already contained in Von Neumann and Morgenstern (1953), see Sect. 17.3, but the authors of that book had not followed it through. Nash assumes that a satisfactory solution of a game exists. He further assumes that the solution is unique. Hence, the solution takes the form of a (mixed) strategy profile  $\sigma$ , recommending to each player  $i$  a course of behavior  $\sigma_i$ . Rational players will know this solution and a player will be willing to behave according to it only if it is in his interest to do so, that is if the strategy  $\sigma_i$  is a best response against the strategy profile of the opponents,  $u_i(\sigma) = \max_{\tau_i} u_i(\tau_i, \sigma_{-i})$ . A *Nash equilibrium* is a profile of strategies  $\sigma$  such that this condition is satisfied for each player  $i$ . To summarize the argument, a necessary condition for  $\sigma$  to be a satisfactory solution of the game is that  $\sigma$  be a Nash equilibrium.

By relying on fixed point theorems (either the one of Brouwer or the one of Kakutani can be used), Nash proved that every finite game admits at least one Nash equilibrium, albeit possibly in mixed strategies. Indeed, in "matching pennies" each player should choose each side of the coin with 50% probability in the unique equilibrium. In a 2-person zero-sum game,  $\sigma$  is a Nash equilibrium if and only if it is a pair of optimal strategies, hence, Nash's concept generalizes the solution provided by Von Neumann and Morgenstern.

Pursuing the above rationale leading to Nash's solution concept, three questions remain: (i) why does a player represent his uncertainty about the behavior of his opponents by an independent mixed strategy profile? (ii) do the requirements determine a unique solution? and (iii) is being a Nash equilibrium sufficient to qualify as a satisfactory solution? The first issue is taken up in Aumann (1974) where an alternative concept of *correlated equilibrium* is developed that allows for more general beliefs. One interpretation is that players discuss before the game and construct a common randomization device, a correlated strategy  $\sigma$ . Upon hearing what pure strategy the device selected for player  $i$ , this player updates his beliefs on his opponents' actions;  $\sigma$  is a correlated equilibrium if each player is always willing to follow the recommendation of  $\sigma$ . The answer to the second question is a clear no, it is easy to construct games with multiple Nash equilibria and many games, also some with practical relevance, do have multiple equilibria. Hence, the rationale that has been given for the equilibrium concept appears incomplete, and the question is, whether, in a game with multiple equilibria there is any argument for focusing on any of these. Alternatively, should one look in an entirely different direction for a satisfactory solution, for example, by giving up the assumption that the solution be unique? We discuss these questions below after first having taken up the third question and having shown that that one also has a negative answer: not every Nash equilibrium can be considered as a satisfactory solution.



Consider the following extensive form *ultimatum game*. Player 1 divides \$10 in any (integer) way that he wants; player 2 observes the division and decides whether or not to accept it; if player 2 accepts, each player receives the amount that was proposed, otherwise no player receives anything. In addition, assume that each player cares only about the amount of money that he receives and that each prefers more money to less, say  $u_i(x) = x_i$ . Player 1 then knows that player 2 is sure to accept as long as he is offered at least \$1, hence, there seem only two possible solutions of the game  $\langle 10; \text{accept all} \rangle$ , and  $\langle 9, \text{accept iff } x_2 \geq 1 \rangle$ . (The first number is the amount that player 1 asks for himself). Indeed these two strategy profiles are Nash equilibria, however, there are other equilibria as well, for example  $\langle 3, \text{accept iff } x_2 \geq 7 \rangle$  is a Nash equilibrium. (If player 1 demands 3 for himself, then player 2's strategy prescribes to accept and that is a best response; on the other hand, if player 2 indeed accepts only if he is offered at least 7, then it is optimal for player 1 to offer him exactly 7). In this equilibrium player 2 "threatens" to reject a positive amount that is offered to him, such as \$5, even though he strictly prefers to accept in case \$5 would be offered. The reason the profile is in equilibrium is that an amount such as \$5 is not offered and, hence, the threat is not called. Player 1 behaves as if player 2 is committed to his strategy, in the extensive form game, however, such commitments are not possible and facing the fait accompli that \$5 is offered, player 2 will accept. Player 1 can neglect the incredible threat.

Starting with Selten (1965) a literature developed dealing with the question of how to eliminate equilibria involving incredible threats. Selten (1965) proposed the concept of *subgame perfect equilibrium* that is based on the idea of "persistent rationality": everywhere in the game tree, no matter what happened in the past, the player that moves will play a strategy that, from that point on, is a best response against the strategies of the opponents. Formally, a subgame perfect equilibrium is a strategy profile that constitutes a Nash equilibrium in every subgame of the game, a subgame being a part of the game that constitutes a game in itself. For games with perfect information, subgame perfect equilibria can be found by the backward induction (dynamic programming) procedure that was already used in Zermelo (1913) to show that the game of chess is completely determined: starting at the end of the game one works backwards each time reducing the game by substituting a decision set of a player by an optimal decision and its associated payoff. It is worthwhile to remark that this assumption of "persistent rationality" was already criticized in Von Neumann and Morgenstern (1944).

Selten (1976) noted that requiring subgame perfection was not sufficient to eliminate all equilibria involving incredible threats and proposed a different solution. He reasoned that, if the problem is caused by non-maximizing behavior at unreached information sets, it can be eliminated by ensuring that all information sets are reached, if only with small probability. Formally, Selten (1976) defines an equilibrium to be (*trembling hand*) *perfect* if it satisfies a robustness test: each player should be still willing to play his equilibrium strategy if each player with some small probability makes mistakes and, as a consequence, every information set is

reached with positive probability. In the ultimatum game above, if player 1 makes mistakes, player 2 is forced to accept any positive amount and player 1 will not offer more than \$1 to player 2 in a perfect equilibrium. The robustness test proposed is weak (the strategies are only required to be optimal for some sequence of small mistakes that converges to zero) and this guarantees that each game admits at least one perfect equilibrium. However, at the same time this implies that some equilibria are perfect only because of the fact that the mistake sequence can be artificially chosen, hence, the perfectness concept does not eliminate all equilibria that are considered as "counterintuitive". The same remark applies to the closely related concept of *sequential equilibria* that has been proposed in Kreps and Wilson (1982). This concept requires that, at each information set, a player constructs beliefs over where he is in the set and optimizes his payoff, given these beliefs and taking the strategies of the opponents as fixed. Kreps and Wilson require that the beliefs be explainable in terms of small mistakes, hence, the close connection between their concept and Selten's.

Intuitively, one would like an equilibrium not to be robust against just some trembles but rather against "plausible" ones: if there is a good explanation for why the information set is reached after all, then preference should be given to such an explanation. Plausibility is hard to define formally, but, if an equilibrium is robust against all trembles, then clearly it will also be robust against the plausible ones. Unfortunately, equilibria satisfying this strict robustness test need not exist. However, Kohlberg and Mertens (1984) have shown that sets of equilibria with this desirable property exist always. They, furthermore, argue that it is natural to look at sets of "equivalent" equilibria. For example, if a player is fully indifferent between two of his strategies, there is no reason to make him choose a specific one. Similarly, if the choice only makes a difference if and after another player has played a strictly dominated strategy, one might be satisfied to leave the choice undetermined.

Sets of stable equilibria satisfy several desirable properties, such as being robust against elimination of strategies that are dominated or that are not a best response against any element of the set. The latter property is also called "forward induction" and it has proved powerful to reduce the number of equilibrium outcomes in signaling games, see the entry on the ECONOMICS OF INFORMATION. The original stability concept proposed in Kohlberg and Mertens (1984) was of a preliminary nature and was not completely satisfactory. The ideas have been further developed in Mertens (1989) but that concept has not seen too many applications yet as it is difficult to handle. In addition, part of the profession considers this a too demanding rationality concept. We refer to Van Damme (1991) for further discussion.

The normative, "rationalistic" interpretation of Nash equilibrium that has been considered thus far relies on the assumption that the solution to a game is unique, yet many games admit multiple equilibria, even multiple stable ones. The question thus arises about equilibrium selection: how will players coordinate on an equilibrium and on which one?

Consider the following simple normal form *stag hunt game* as an example. Two players simultaneously choose between the numbers 1 and 2. If both choose the same number, they each receive a payoff equal to the number chosen, if they choose different numbers the player choosing the lower number receives 1 from the other. The game has two equilibria in pure strategies: (1,1) and (2,2). The latter yields both players higher payoffs than the former, however, choosing 2 is also more risky than choosing 1: while the latter guarantees the payoff 1, the former might result in a loss of 1. How to resolve the problem?

Harsanyi and Selten (1988) provides a general theory of equilibrium selection in non-cooperative games that can be used in these circumstances. The theory can be seen as an extension of Nash's (1953) equilibrium selection theory for bargaining games. The authors start by formulating general properties that a theory should satisfy (such as symmetry, ordinality, efficiency, monotonicity, consistency) but they quickly discover that not all of these can be satisfied at the same time. Indeed, the stag hunt game given in the above paragraph has the same best reply structure as the game in which payoffs are 2 (resp. 1) if each player chooses 1 (resp. 2) and where payoffs are 0 if different numbers are chosen, and in this game the efficient equilibrium involves both players choosing 1. Hence, choices have to be made and, as Harsanyi and Selten admit, different selection theories are possible. One of the major innovations of Harsanyi and Selten (that in some form will probably play a role in these alternative theories as well) is the concept of *risk dominance*. Intuitively, one equilibrium is said to risk dominate another if, in a situation where attention is confined to this pair, players eventually come to coordinate on the first as they consider the second to be more risky. The formal definition makes use of the *tracing procedure*, an algorithm (homotopy) that transforms any mixed strategy profile (which represents the players' initial assessment about how the game will be played) into an equilibrium of the game. This tracing procedure is supposed to capture the reasoning process of rational players in the game. If, in a situation of uncertainty about equilibria  $\sigma$  and  $\sigma'$ , the tracing procedure produces equilibrium  $\sigma$ , then  $\sigma$  is said to risk dominate  $\sigma'$ . The concept allows a simple characterization for 2-player 2 x 2 games with 2 strict equilibria. Let  $p_i(\sigma, \sigma')$  be the probability that player  $i$  has to assign to  $\sigma_i$  in order to make his opponent  $j$  indifferent between  $\sigma_j$  and  $\sigma'_j$ . Intuitively, the larger  $p_i$ , the more attractive  $\sigma'_j$  is for player  $j$ . In this case,  $\sigma$  risk dominates  $\sigma'$  if and only if  $p_1(\sigma, \sigma') + p_2(\sigma, \sigma') < 1$ . In the above example, the equilibrium (1,1) risk dominates the equilibrium (2,2).

### 3. Evolution and Learning

The above discussion of Nash equilibrium (and its variants) has been based entirely on the rationalistic interpretation of this concept, viz. that a self-enforcing theory advising players how to play should prescribe a Nash equilibrium in every game. In his PhD-Thesis (Nash, 1950), John Nash, however, provided also a second interpretation of his concept. When the game is played repeatedly by myopic players who best respond against the current situation, then, if a stable state is reached, that state has to be a Nash equilibrium. In this second interpretation it is thus unnecessary to assume that players know the full structure of the game, or are able to perform complex reasoning processes. On the contrary, the interpretation applies in a context of limited information where players behave in a boundedly rational way. After Shapley (1964) had published an example showing that a certain dynamic adjustment process need not converge at all, interest in this second approach dwindled for some time, but it was revived in the 1990s. Since then a systematic study has begun under which circumstances a variety of processes will converge and, if they converge, to which equilibria. It is too early to draw broad conclusions, instead the reader is referred to Fudenberg and Levine (1998) for a partial overview. Interestingly, some of the concepts that were introduced in the 'rationalistic' branch reappear in this branch of the literature, in particular this holds for the concept of risk dominance.

Consider the "Stag Hunt" coordination game described above and imagine that it is played repeatedly. When a player is called upon to play he "asks around" to find out what other players in his population that have already played have been confronted with and he plays a best response against the resulting sample. Young (1998) shows that the process will converge to one of the two equilibria. Now imagine that sometimes players make mistakes in implementing their desired action and choose the other one instead. In this case the process may move from one equilibrium to the other through a sequence of consecutive mistakes. Young (1998), however, shows that, if the probability of making mistakes tends to zero, the limit distribution will put all mass on the risk dominant equilibrium as that equilibrium is more robust to mistakes than the other one. Hence, in the very long run when mistake probabilities are negligible the process will end up in the risk dominant equilibrium, irrespective where it starts out. The process not only produces an equilibrium, it leads to a very specific one. Research effort is under way to investigate the robustness of this result with respect to both the underlying game and the dynamic process under consideration.

Related to the above is the application of the Nash equilibrium concept in biology. As Maynard Smith (1982, p. vii) writes "Paradoxically, it has turned out that game theory is more readily applied to biology than to the field of economic behavior for which it was originally designed". In biological applications the concept of utility is replaced by that of Darwinian fitness (expected number of offspring), human rationality is replaced by evolutionary stability and, according to Maynard Smith, both contribute to making the theory more applicable. Note

that in this context it is assumed that there is no conscious choice of strategy at all; strategies exist and individuals are "programmed" to play certain strategies, an evolutionary process selects those strategies that have highest fitness and in the end an equilibrium results.

To illustrate, one may consider the so called "Hawk-Dove" game in which two animals are contesting a certain valuable resource. There is a (finite) repertoire of possible behaviors,  $S$ , and when an individual programmed to play strategy  $s$  meets another individual playing strategy  $s'$  the gain in fitness to the first is given by  $u(s, s')$ . The situation may then be represented by a game in which there are 2 players, and each has strategy set  $S$  and payoff function  $u$ , i.e., the game is symmetric. The question is which strategy will be selected, i.e., which one will survive in the long run. Obviously, if  $u(s', s) > u(s, s)$  then  $s'$  can displace  $s$  so that a necessary condition for survival of  $s$  is that  $(s, s)$  constitutes a Nash equilibrium of the game. A second necessary condition for selection of  $s$  is that, once  $s$  has been established, random mutations that enter into the population are selected against, i.e., these are driven out again. This will be the case if they cannot spread, i.e., if the entrant  $s'$  is an alternative best response against  $s$ , then the incumbent strategy obtains a higher payoff against the mutant than the mutant gets against itself (if  $u(s', s) = u(s, s)$ , then  $u(s, s') > u(s', s')$ ). A strategy  $s$  satisfying both conditions is called an *evolutionarily stable strategy* or ESS. One sees that an ESS corresponds to a Nash equilibrium satisfying an additional stability requirement. We refer to Hammerstein and Selten (1994) for further discussion.

## BEHAVIOR

Game theory, at least its traditional variant, relies on the assumption of rational behavior, that is, behavior that is consistent and is motivated solely by the desire to obtain well defined goals, with, furthermore, no cognitive constraints standing in the way. In contrast, the biological branch of the theory relies on no rationality at all, but assumes long time periods for selection to work. The question, hence, is how the theory is relevant for humans that are boundedly rational and have relatively little time to learn and adjust behavior. Real players have only limited rationality: they face cognitive limitations which may make it impossible to get a full overview of the situation and to consistently evaluate and rank all alternatives, they may decide not to think and instead rely on automated procedures that have proved satisfactory in the past or in other related contexts, and even if rational deliberation suggests a decision, emotional factors may override and produce an alternative decision. The question, hence, is to what extent a theory that is based on strong rationality assumptions is relevant for actual interactive decision making. For sure, a normative solution can serve as a benchmark to better understand actual behavior, it might point to systematic differences and it might even be that the differences are negligible in certain contexts, such as when players have had sufficient time to familiarize themselves with the situation. Unfortunately, a discussion of the issue of relevance is made difficult by the fact that alternative theories are still undeveloped and since relatively little is known yet about the actual

structure of human decision making, see Selten (1989). Nevertheless, some systematic deviations from the theory that are found in the experimental laboratory may be described. See Camerer (1997) for more details.

First of all, a remark that does not apply that much to the solution concepts, but rather to the modeling aspect. Game theory analyses the consequences of selfish behavior, where selfishness is interpreted as the individual following his preferences. However, it is not necessarily assumed that individuals are selfish in the more narrow sense of being only interested in their own material well-being. The latter assumption would indeed be too narrow. Experiments have shown that, in the ultimatum game referred to above, responding players generally dislike being treated "unfairly", hence, they are willing to reject proposals in which they get positive amounts but the distribution is uneven. A relevant game model of the situation should take this into account and as a consequence it may well advise to player 1 to allocate a substantial proportion of the cake to player 2. The reader is referred to Kagel and Roth (1995) for further discussion on this issue. Suffice it to note here that tests of game theoretic predictions are always combined tests of the underlying game and the solution concept applied, that it is difficult to control the players preferences, hence, that one should be careful in drawing strong inferences from results observed in the experimental laboratory.

Secondly, when confronted with an interactive decision situation, real life players typically construct a simplified model of the situation. The "mental model" may be an oversimplification and, as a consequence, important strategic aspects may be missed. Furthermore, a player need take into account the models constructed by other players: if these do not incorporate certain elements, it does not make sense for the player to signal along these dimensions. Selten (1998) argues that the reliance on superficial analysis may explain presentation or framing effects, that is, the way in which a game is presented to the players may have a strong influence on actual behavior. The basic reason is that the presentation may trigger a certain reasoning process, or may make some outcomes more focal than others. Related is the fact that human players do not analyse each game in isolation but rather make use of analogies; a principle that has proved useful in a certain context may also be used in another one. In experiments, for example, it is observed that 2-player games with sequential moves in which the second mover receives no information about the first move may be played as if there was such information, where according to the theory the game is strategically equivalent to the game with simultaneous moves. The discussion of the presentation effect in strategic contexts originates with Thomas Schelling (see Schelling (1960)), but his concept of "focal points" still largely awaits formalization.

Where standard game theory assumes that all players are equally rational and, hence, homogeneous, experiments have revealed considerable player heterogeneity. Some players are more of the adaptive type and do not approach a game analytically. Instead, in repeated contexts,

they make use of ex-post rationality considerations and move in the direction of better responses against the previous outcome. Players that rely on analytic approaches construct simplified models, as discussed above, and they may limit the "depth" of their analysis to a couple of steps and, hence, may not reach a game theoretic equilibrium. For example, in the "beauty contest game", in which a set of players choose numbers between 1 and 100 and in which the winner is the one that chooses the number closest to half of the average of all numbers, the equilibrium is to choose 1 and that equilibrium is reached through iterative elimination of dominated strategies, however, most players choose larger numbers and those that choose the equilibrium number do not win the game. Relatedly, in extensive form games players do not necessarily "look ahead and reason backwards", but instead may use a myopic somewhat forward looking process and, as a consequence, the outcome obtained may be different from the subgame perfect equilibrium one.

At present research is under way to construct an empirically based behavioral game theory that steers a middle ground between over-rational equilibrium analyses and under-rational adaptive analyses, but it is too early to survey this field. Also see the entries on BEHAVIORAL ECONOMICS, DECISION-MAKING UNDER RISK AND UNCERTAINTY and EXPERIMENTAL ECONOMICS.

## CONCLUSION

The space allotted has not allowed to discuss any applications in depth, however, as Aumann (1987) has forcefully argued, the models and solution concepts from game theory should be judged by the insights that they yield. In addition to the entries of this Encyclopedia already referred to above, we may point to the areas of finance (market micro structure), industrial organization and antitrust and regulation, in which ideas from non-cooperative game theory have been influential and are widely used.

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